IPP-SR-6: General covariance

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HT25

The course

1. Newton's laws
2. Galilean invariance
3. The Michelson-Morley experiment
4. Einstein's 1905 derivation of the Lorentz transformations
5. Spacetime structure
6. General covariance
7. Relativity and conventionality of simultaneity
8. Frame-dependent effects
9. The twin paradox
10. Dynamical and geometrical approaches to relativity
11. Presentism and relativity
12. Acceleration and redshift

If only I knew more mathematics! (Schrödinger, 1925)

Today

Physical laws

General covariance

Kleinian and Riemannian conceptions of geometry

What is special relativity?

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Laws in index notation

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- Let us now consider how to write some familiar physical laws using this index notation.

Example 1: Klein-Gordon equation

$$-\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}+\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}=0$$

$$\begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial t} & \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{pmatrix} \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial t} \\ \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \phi = 0$$

$$\eta_{\mu\nu}\partial^\mu\partial^\nu\phi = 0.$$

Example 2: Newton-Poisson equation

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}=4\pi\rho$$

$$\begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial t} & \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{pmatrix} \begin{pmatrix} 0 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{c}\frac{\partial}{\partial t} \\ \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \phi = 4\pi\rho$$

$$h^{\mu\nu}\partial_\mu\partial_\nu\phi=4\pi\rho.$$

Example 3: Maxwell's equations

$$
\begin{aligned}
\nabla \cdot \mathbf{E} &= \rho \\
\nabla \cdot \mathbf{B} &= 0 \\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
\nabla \times \mathbf{B} &= \mathbf{J} + \frac{\partial \mathbf{E}}{\partial t}
\end{aligned}
$$

Example 3: Maxwell's equations

$$
F^{\mu\nu} = \begin{pmatrix} 0 & -E_1/c & -E_2/c & -E_3/c \\ E_1/c & 0 & -B_3 & B_2 \\ E_2/c & B_3 & 0 & -B_1 \\ E_3/c & -B_2 & B_1 & 0 \end{pmatrix},
$$

$$
J^\mu = \begin{pmatrix} \rho \\ J^i \end{pmatrix}.
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$$J^{\mu} = \begin{pmatrix} \rho \\ J^i \end{pmatrix}.$$

Then Maxwell's equations can be written:

$$\eta_{\mu\lambda}\partial^{\lambda} F^{\mu\nu} = J^{\nu},$$

$$\partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} + \partial_\lambda F_{\mu\nu} =: \partial_{[\mu} F_{\nu\lambda]} = 0.$$

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\end{aligned}
$$

(**Exercise:** Plug in components into the above two equations in order to derive Maxwell's equations in their 3-vector forms.)

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- These are just the Lorentz transformations!
(**Exercise:** show this.)
- The equations are also invariant under translations, making them invariant under the full Poincaré group.
- One sometimes hears the claim that writing a theory using four-dimensional indices makes the symmetries of one's equations ‘manifest’.

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- The simplest form of this equation will be preserved under coordinate transformations which preserve the diagonal form of $h^{\mu\nu}$, i.e. coordinate transformations such that $M^{\mu}{}_{\sigma} M^{\nu}{}_{\lambda} h^{\sigma\lambda} = h^{\mu\nu}$.
- Assuming that the transformations are linear, these are just the Galilean transformations![1] (Once we also include translations.) (**Exercise:** Show this.)

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- In fact, the index notation makes it pretty easy to transform to an arbitrary (rather than inertial) coordinate system, and see these equations in their general (and ugly!) form.
- (Recall from lecture 1 N2L in an arbitrary frame.)

Explicit illustration: Klein-Gordon equation

$$\eta_{\mu\nu}\partial^{\mu}\partial^{\nu}\varphi = 0$$

$$\eta_{\mu\nu}\frac{\partial}{\partial x_{\mu}}\frac{\partial}{\partial x_{\nu}}\varphi = 0$$

$$\longrightarrow \eta_{\mu\nu}\frac{\partial x_{\mu'}}{\partial x_{\mu}}\frac{\partial}{\partial x_{\mu'}}\left(\frac{\partial x_{\nu'}}{\partial x_{\nu}}\frac{\partial}{\partial x_{\nu'}}\varphi\right) = 0$$

$$\eta_{\mu\nu}\frac{\partial x_{\mu'}}{\partial x_{\mu}}\left(\frac{\partial^2 x_{\nu'}}{\partial x_{\mu'}\partial x_{\nu}}\frac{\partial}{\partial x_{\nu'}}\varphi + \frac{\partial x_{\nu'}}{\partial x_{\nu}}\frac{\partial}{\partial x_{\mu'}}\frac{\partial}{\partial x_{\nu'}}\varphi\right) = 0$$

$$\eta_{\mu\nu}\frac{\partial^2 x_{\nu'}}{\partial x_{\mu}\partial x_{\nu}}\partial^{\nu'}\varphi + \eta_{\mu\nu}\frac{\partial x_{\mu'}}{\partial x_{\mu}}\frac{\partial x_{\nu'}}{\partial x_{\nu}}\partial^{\mu'}\partial^{\nu'}\varphi = 0.$$

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\end{aligned}
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Note the extra term in the non-inertial frame (cf. fictitious forces).

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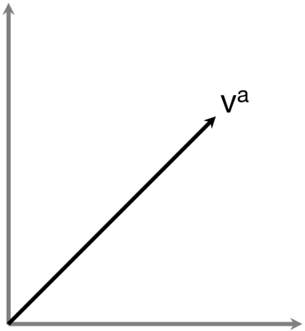
  1. Write its equations in an arbitrary frame, with all extra terms included.
  2. Write the theory in a *coordinate-independent* language.
- We've seen option (1); let's now think a bit more about option (2).

Objects versus components



$$V^\mu = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$$

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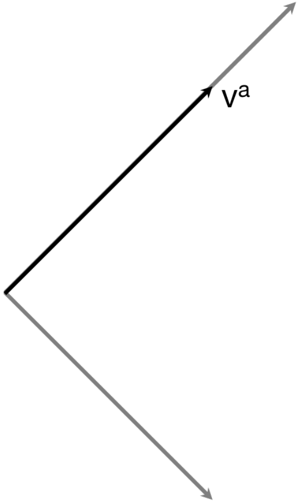

$$V'^{\mu} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Coordinate-independent formulations

- To write a theory in a coordinate-independent way, we move from using *coordinate indices* ($\mu, \nu, \ldots$), which represent the *components* of objects in a particular coordinate basis, to *abstract indices* ($a, b, \ldots$), which directly represent the objects themselves.

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- This involves no reference to a coordinate system at all—so *a fortiori* holds in all coordinate systems.
- The details are beyond the scope of this course, but see e.g. (Friedman 1983) and (Malament 2012) for details.

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- But should this be regarded as an autonomous entity (object in our ontology), or just a *codification* of the symmetries of the coordinate-based dynamical equations from which we began?

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- But should this be regarded as an autonomous entity (object in our ontology), or just a *codification* of the symmetries of the coordinate-based dynamical equations from which we began?
- We will address this issue in lecture 10. (Cf. also lecture 1.)

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Two conceptions of geometry

Kleinian conception: Geometry is characterised via the invariance groups of certain structures under coordinate transformations.

Riemannian conception: Geometry is characterised via metric tensors and similar differential-geometric objects.

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Aristotelian spacetime

$$t \mapsto \pm t + \tau$$
$$\mathbf{x} \mapsto \mathbf{R}\mathbf{x}$$

In Aristotelian spacetime, there is:

1. A notion of spatial distance.
2. A notion of temporal distance.
3. A standard of rotation across time.
4. A notion of straightness of paths across time.
5. A preferred velocity.
6. A preferred point.

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Riemannian approach: $\langle M, t_{ab}, h^{ab}, \nabla_a, \sigma^a, \zeta \rangle$.

Newtonian spacetime

$$t \mapsto \pm t + \tau$$
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Riemannian approach: $\langle M, t_{ab}, h^{ab}, \nabla_a, \sigma^a \rangle$.

Neo-Newtonian/Galilean spacetime

$$t \mapsto \pm t + \tau$$
$$\mathbf{x} \mapsto \mathbf{R}\mathbf{x} + \mathbf{v}t + \mathbf{a}$$

In Neo-Newtonian/Galilean spacetime, there is:

1. A notion of spatial distance.
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4. A notion of straightness of paths across time.
5. ~~A preferred velocity.~~
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Riemannian approach: $\langle M, t_{ab}, h^{ab}, \nabla_a \rangle$.

Maxwellian/Newton-Huygens spacetime

$$t \mapsto \pm t + \tau$$
$$\mathbf{x} \mapsto \mathbf{R}\mathbf{x} + \mathbf{a}(t)$$

In Maxwellian/Newton-Huygens spacetime, there is:

1. A notion of spatial distance.
2. A notion of temporal distance.
3. A standard of rotation across time.
4. ~~A notion of straightness of paths across time.~~
5. ~~A preferred velocity.~~
6. ~~A preferred point.~~

Riemannian approach: $\langle M, t_{ab}, h^{ab}, [\nabla_a] \rangle$.

Leibnizian spacetime

$$t \mapsto \pm t + \tau$$
$$\mathbf{x} \mapsto \mathbf{R}(t) + \mathbf{a}(t)$$

In Leibnizian spacetime, there is:

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5. ~~A preferred velocity.~~
6. ~~A preferred point.~~

Riemannian approach: $\langle M, t_{ab}, h^{ab} \rangle$.

Machian spacetime

$$t \mapsto f(t) \qquad\qquad (f \text{ monotonic})$$
$$\mathbf{x} \mapsto \mathbf{R}(t) + \mathbf{a}(t)$$

In Machian spacetime, there is:

1. A notion of spatial distance.
2. ~~A notion of temporal distance.~~
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Riemannian approach: $\langle M, h^{ab} \rangle$.

Minkowski spacetime

$$x^\mu \mapsto \Lambda^\mu{}_\nu x^\nu + a^\mu \qquad (\Lambda^\mu{}_\nu \in SO(1,3))$$

In Minkowski spacetime, there is:

1. ~~A notion of spatial distance.~~
2. ~~A notion of temporal distance.~~
3. A standard of rotation across time.
4. A notion of straightness of paths across time.
5. ~~A preferred velocity.~~
6. ~~A preferred point.~~
7. A notion of *spacetime* distance.

Riemannian approach: $\langle M, \eta_{ab} \rangle$.

Connections

The groups specified in the Kleinian conception of geometry are those transformations which would leave invariant the structures presented on the Riemannian conception of geometry, if they were to be written in a coordinate basis.

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E.g., write η_{ab} in a coordinate basis, becomes $\eta_{\mu\nu}$; this matrix is preserved under Poincaré transformations.

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2. "Special relativity is the statement that the laws of physics (in standard formulation) are Poincaré invariant."
3. "Special relativity is the statement that spacetime structure (over and above topological and differentiable structure) is exhausted by Minkowski spacetime."

What is special relativity?

At least three possible answers to this question:

1. "Special relativity consists of the RP, the LP, whatever supplementary principles are needed to derive the Lorentz transformations therefrom, and the said derivation of the Lorentz transformations."
2. "Special relativity is the statement that the laws of physics (in standard formulation) are Poincaré invariant."
3. "Special relativity is the statement that spacetime structure (over and above topological and differentiable structure) is exhausted by Minkowski spacetime."

Question: Which of the above captures the 'essence' of special relativity?

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In this lecture, we've:

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1. Seen how to write physical theories in index notation.
2. Seen how to write physical theories in arbitrary frames, using that index notation.
3. Seen (something of) how to write physical theories in a coordinate-independent manner.
4. Witnessed the Riemannian approach to geometry and spacetime structure.
5. Considered the question of the essence of special relativity.

References

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- Michael Friedman, *Foundations of Space-Time Theories*, Princeton, NJ: Princeton University Press, 1983.
- David B. Malament, *Topics in the Foundations of General Relativity and Newtonian Gravitation Theory*, Chicago, IL: University of Chicago Press, 2012.
- Erwin Schrödinger, letter to Wilhelm Wien, 27th December 1925.